NONLINEAR OPTIMISATION FOR VARIOUS APPLICATIONS OF A TIME-DOMAIN DIGITAL CODING METASURFACE

Teo Kai Wen[1], Chia Tse Tong[2]
[1]Raffles Institution, 10 Bishan Street 21, Singapore 574013
[2]DSO National Laboratories, 12 Science Park Drive, Singapore 118225

Abstract

Metasurfaces hold much potential in communication infrastructure, making use of controllable reflection to modify the diffraction pattern of the reflective array. In this project, the efficacy of different methods of nonlinear optimisation for a newly developed time-domain digital coding metasurface [1] is investigated, and its various applications are discussed. Specifically, a genetic algorithm, particle swarm optimisation and integer particle swarm optimisation were used to optimise the metasurface for applications such as harmonic beam steering with side lobe suppression, diffusion and dual-harmonic beam steering. It was found that integer (discrete) particle swarm optimisation was most suitable to optimise this metasurface for harmonic beam steering, consistently finding better solutions than the other methods. This method is hence used to optimise the metasurfaces for other applications such as diffusion and dual-harmonic beam steering. The potential of using this unusual coding technique for such applications is also discussed.

Introduction

A time-domain digital coding metasurface is a surface consisting of discrete reflectors, each of which has a time-dependent reflection coefficient that can be controlled. This reflection coefficient is a complex number, as it can change the phase and amplitude of incident light. With the ability to control the reflection coefficient in both space and time, the metasurface exhibits unique properties.

This project is based on the work of Chen [1], and uses the same coding method for the metasurface, with the individual elements switching between two reflection coefficients Γ_1 and Γ_2 periodically, with a period of TT. The fraction of the period taken up by the first reflection coefficient is known as the duty ratio $MM \equiv \tau\tau/TT_0$, which varies between 0 to 1. The time delay tt_0 refers to the phase of the cycle that it begins at. The time delay and duty ratio of each coding element can be varied individually.

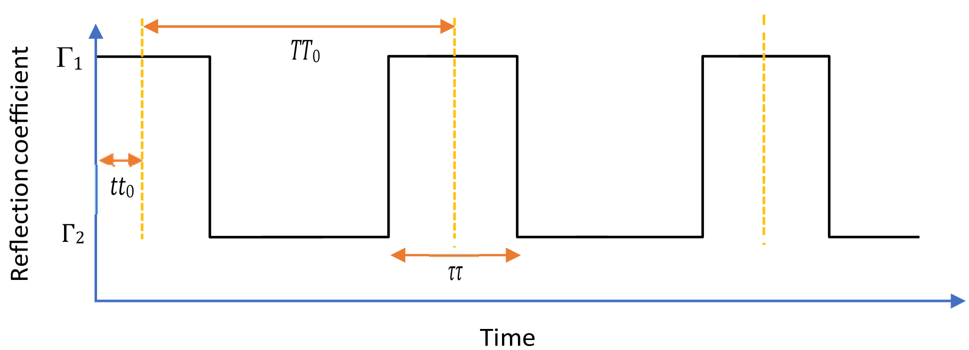

Figure 1: Variance of reflection coefficient over time

The far-field reflection of the metasurface can be calculated as such. For an element of the metasurface at column p and row q (both 0-indexed for convenience), the phasor representing the reflected wave f_{pq} is

$$f_{pq} = E_{pq}(\theta)\Gamma_{pq}(t)\exp\left(2\pi j\frac{\mathbf{r}_{pq}\cdot\hat{\mathbf{p}}}{\lambda_c}\right)$$

where E_{pq} is the element scattering pattern of the pq metasurface element, $\Gamma_{pq}(t)$ is the (complex) reflection coefficient as a function of time, $E(\theta) = \sqrt{\cos\theta}$ is the element pattern (Lambertian reflection), $\mathbf{r}_{pq}$ is the position vector of the element relative to, $\hat{\mathbf{p}}$ is the unit vector pointing from the element to the observer and λ_c is the frequency of the incident light (normal to the plane)

Expanding the dot product and using spherical coordinates, the total far-field reflected wave is

$$f(\theta,\phi,t) = \sum_{q=0}^{N-1}\sum_{p=0}^{M-1} E_{pq}(\theta)\Gamma_{pq}(t)\exp\left(\frac{2\pi j}{\lambda_c}(p p_x \sin\theta\cos\phi + q p_y \sin\theta\sin\phi)\right)$$

Here, we have defined the array to be in the $x-y$ plane. p_x is the element spacing in the x-direction and p_y is the element spacing in the y-direction. $j \equiv \sqrt{-1}$. By convention (ISO 80000-2:2019), θ is the angle from the z-axis, and ϕ is the azimuthal angle, defined positive anticlockwise from the x-axis. There are a total of M columns and N rows.

Expanding into a Fourier series,

$$f(\theta,\phi,k) = \sum_{q=0}^{N-1}\sum_{p=0}^{M-1} E_{pq}(\theta)\exp\left(\frac{2\pi j}{\lambda_c}(p p_x \sin\theta\cos\phi + q p_y \sin\theta\sin\phi)\right) a^k_{pq}$$

the Fourier series coefficients are

$$a_k = \begin{cases} M\cdot\Gamma_1 + (1-M)\cdot\Gamma_2, & k = 0 \\ r_0 M\,|\mathrm{sinc}(k\pi M)|\,e^{-j\{k\omega_0 t_0 + \frac{\pi}{2}[1-(-1)^{\lfloor |k|M\rfloor}]\}}, & k\in\mathbb{Z}\setminus\{0\}\end{cases}$$

where $\omega_0 \equiv 2\pi/T_0$, and $r_0 \equiv \Gamma_1 - \Gamma_2$. This is derived in the appendix.

With apt selection of the value ranges of M and t_0, we can control the amplitude and phase of nonfundamental harmonics separately.

$$a_k = r_0 M\,\mathrm{sinc}(k\pi M)\,e^{-jk\omega_0 t_0}, \qquad M\in\left[0,\frac{1}{2|k|}\right],\ t_0\in\left[0,\frac{T_0}{|k|}\right]$$

Within this range, notice that changing the duty ratio only changes the amplitude of the coefficient. Similarly changing the time delay only changes the phase of the Fourier series coefficient.

This allows certain harmonics to be selectively suppressed. For example, if $M = 1/2$, then clearly all even harmonics (i.e. k is even) is suppressed. Other rational values of M can similarly suppress harmonics periodically. This holds much potential in various applications of metasurfaces, such as harmonic beam steering and diffusion, which we will explore in this paper.

Materials and Methods

First, we replicated the results from the paper. In the paper, the duty ratio of the various elements were kept at M=0.5, while the time delay was adjusted in groups of 4 rows (totalling 48 rows, or 12 controllable quantities). The solid dark blue lines in Figure 2 shows the first harmonic viewed in the $y - z$ plane as predicted by the replicated model of the metasurface, and the dotted dark blue line is the theoretically predicted pattern from [1] for f_c= 27 GHz, f_0=100 kHz. We noted that $E_{pq}(\theta)$ has to be $\sqrt{\cos\theta}$ instead of $\cos\theta$ as used in other papers by the same authors [2] The phase distributions for 2(a) and 2(b) are 554433221100 and 963096309630 respectively. The integers refer to the 12 groups' time delays as a fraction of the period. So 5 would mean a time delay of 5T/12. Figure 3 has the same incident frequency.

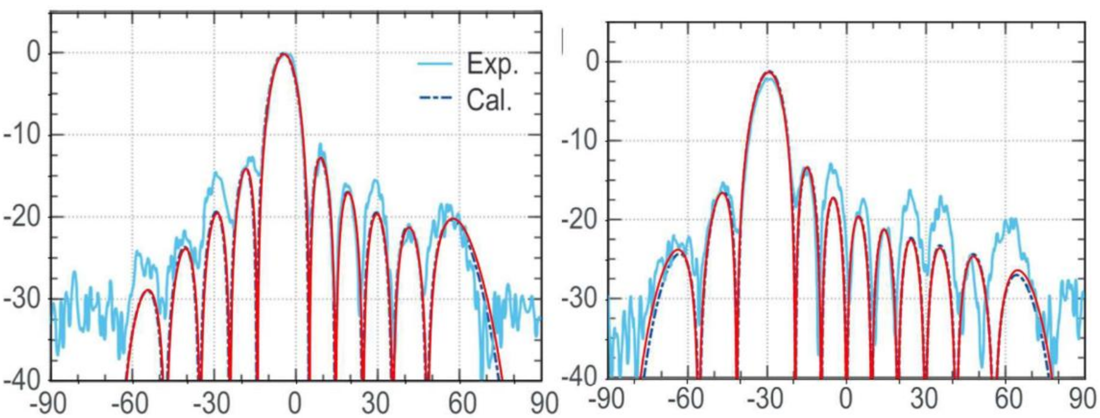

Figure 2: First harmonic radiation pattern for time delay for (a) 554433221100, (b) 963096309630. Solid red line is replicated pattern, dark blue dotted line is the original pattern. Ignore the light blue line; that is the experimental pattern obtained in [1].

With sufficiently accurate replication, we first optimise the metasurface for harmonic beam steering, by varying only the time delay across the 12 groups of rows. To decide an algorithm, we compared the results using a genetic algorithm, continuous particle swarm optimisation and integer particle swarm optimisation. Each algorithm was given the same parameters of 80 particles and 100 generations. The genetic algorithm was not given any crossover between parents as that has little meaning in this application. Instead, the genetic algorithm works through mutations, with 25% of genes randomly mutating. Meanwhile, both PSO algorithms were given identical parameters for the inertial weight, the cognitive and the social parameter. The inertial weight was 0.8, while the cognitive and social parameters were both 0.5.

The continuous PSO and the genetic algorithm were implemented with popular Python libraries, namely scikit-opt and pyGAD respectively. However, the integer PSO, being quite a niche algorithm, was coded by hand.

The fitness function was defined as such:

$$fitness = w * peakratio + (1 - w) * \sin^2\left(peakangle - targetangle + \frac{\pi}{2}\right)$$

Here, w is between 0 and 1. The peak ratio is the ratio of the amplitude of the primary maxima to the next highest maxima. The peak angle is the angle of the primary maxima. Higher values of w incentivises the ratio to be higher while neglecting the target angle more.

The seemingly unusual sine function is intended to incentivise the angle to be close to the target angle, but does not have to be perfectly equal, as the sine function gets less steep close to the target angle. The sine function is squared to make this gradient a little steeper.

It turns out that the IPSO was the best algorithm, as will be elaborated in the results section. We will hence use the IPSO for all other applications explored. First, we tried diffusion: where the energy is uniformly spread out across the angles. This is quantified by simply taking the fitness function to be the reciprocal of the highest peak of the harmonics of concern.

$$ff\pi\pi ttffeeffff = 1/ppeeaakkppee\pi\pi pp\boldsymbol{h}tt$$

We have also attempted to do a dual-harmonic beam steering, where the primary maxima in the different harmonics will be steered to different angles. Similar to harmonic beam steering, the fitness function is simply multiplied for each of the two harmonics in consideration. We have tested with adding the fitnesses of each harmonic, but that tended to find solutions with poor steering precision as it would be overpowered by a good solution in the other harmonic.

Experimental Verification

We have verified that the harmonic beam amplitudes predicted are right with a different metasurface. This is a 8-element 1-dimensional metasurface. However, the harmonic amplitude ratios should be equal. With a duty ratio of M=0.5, there should be only odd harmonics. We normalise the intensities using the first harmonic.

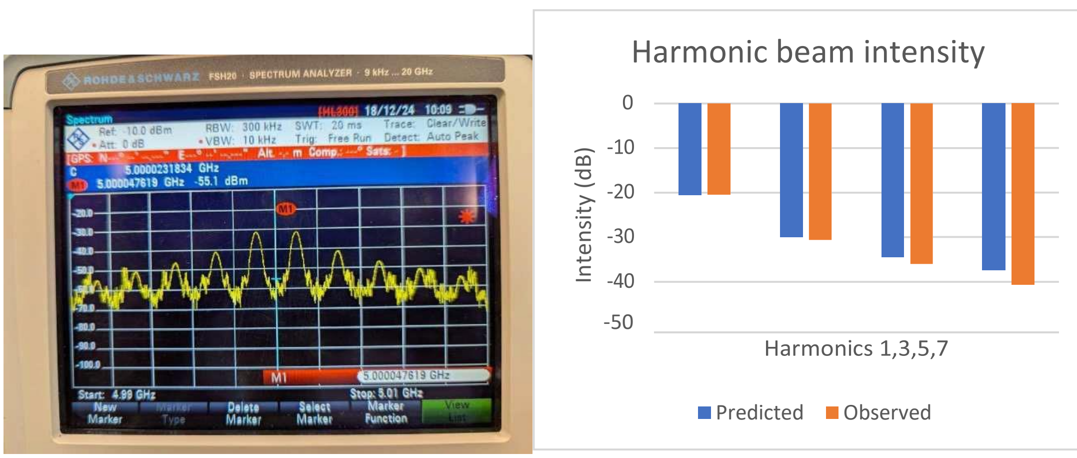

Figure 3: Experimental verification

We can confirm that the harmonic beam intensities are roughly as predicted. There is a lot of noise in the higher order harmonics so the intensities do not align exactly.

Results

For harmonic beam steering, we used a fitness function with a weight of $ww = 1/11$. The algorithm that could find the best solution on average (highest fitness) was the IPSO (see Figure), followed closely by the PSO. It is likely due to the fact that random fluctuations in velocity forces the particle into a significantly different solution and hence allows it to "escape" from the local minimum. The velocities in the continuous PSO may be insufficient for it to escape the local minimum, which explains why the fitness of the best solution tends to plateau after a few generations.

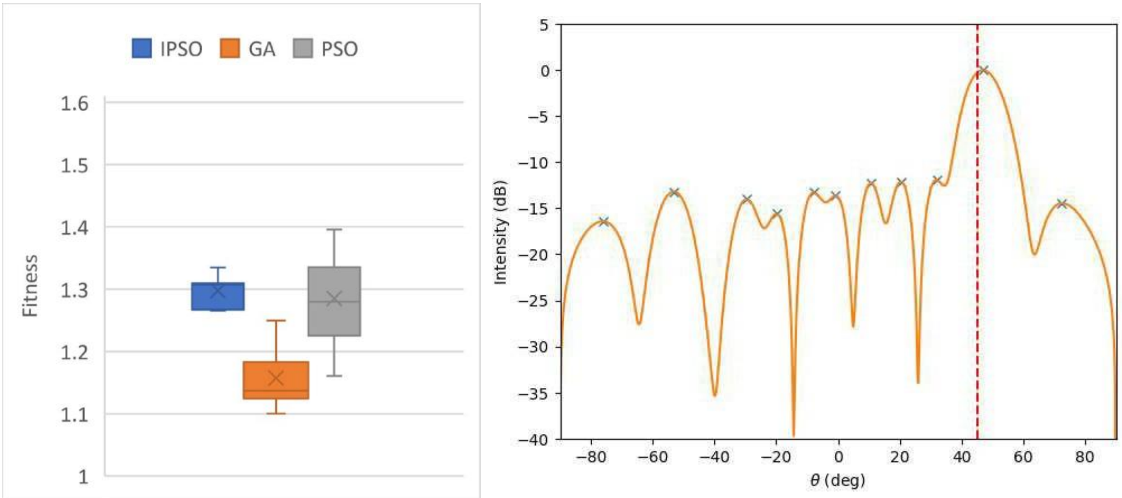

Figure 4: (a) Comparison of optimisation methods for harmonic beam steering, (b) Example of a solution found by the algorithm

The GA does not benefit from any sort of "group" intelligence, so it is practically a bunch of particles performing a random walk through phase space. This is inevitable as crossovers have little meaning in a metasurface. Splicing together the setting of multiple high fitness "parents" does not lead to a high fitness "child".

Curiously, the PSO exhibits much more variability than the other two algorithms. This is possibly due to the tendency for the PSO to find local minima and be unable to escape them. Hence, the performance of the PSO is highly dependent on the initial population. The IPSO is better able to escape local minima, but its discretised nature means that it is unable to find a "perfect" solution. Realistically, much precision is required to achieve the sub-integer accuracy of the solution provided by the PSO, so it may be more feasible to use the nice integer solution provided by the IPSO.

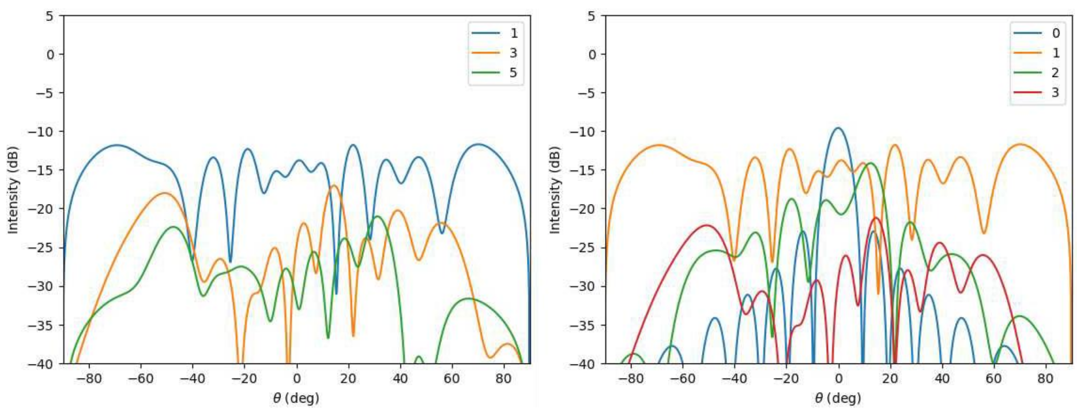

Figure 4: Diffused diffraction pattern for various harmonics with (a) M=0.5 and (b) M=0.4. Time delays are 4,5,2,8,2,8,3,9,4,10,1,6

As for diffusion, we first set M=0.5, as seen in Figure 4(a). It then sufficed to consider only the first harmonic, as the sinc function present in the fourier series coefficient naturally suppresses higher order harmonics. To show how much the energy is diffused, 0 dB is defined to be the highest peak of the corresponding metasurface with the same duty ratio of 0.5 and no time delay. The intensity of the reflected waves was quite evenly distributed in the first harmonic at about -12dB. All the other harmonics were comfortably below the first harmonic.

So far, the duty ratio has been kept at 0.5, with the purpose of suppressing the fundamental frequency. However, if we choose something slightly lower like MM = 0.4 as seen in Figure 4(b), while using the same solution found for MM = 0.5, the harmonic beam amplitudes will be diffused much more, as they are split up among the other harmonics more (though it does restore the fundamental frequency). Within each harmonic, the normalised intensity pattern is independent of MM. We are using the +1 harmonic to normalise the intensity, so the pattern for +1 harmonic remains identical in both Figure 4(a) and Figure 4(b). Note that the symmetrical pattern of the fundamental frequency is due to its independence of the time delay in the fundamental diffraction pattern. So changing the time delay or duty ratio will not change the diffraction pattern of the fundamental frequency.

We have also attempted dual-harmonic beam steering using the IPSO, with the +1 harmonic steered to +45° and the +3 harmonic being steered to -45°. This is using a weight of 0.05. From Figure 5(a), the performance in dual beam steering is noticeably poorer than single beam steering, as expected, since there are more objectives to be met here.

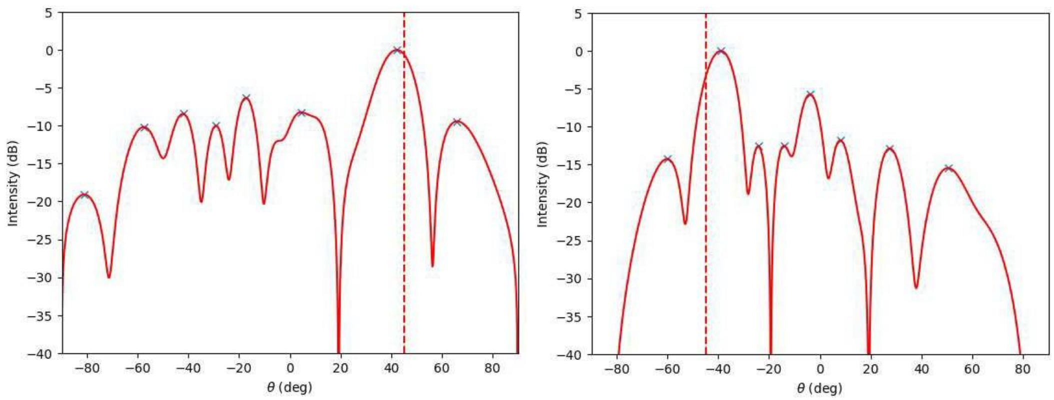

Figure 5: (a): Diffraction pattern of first harmonic (steered to 45°), (b): Third harmonic (steered to -45°), weight = 0.05

Note that it is "easier" to steer to lower angles, as it is closer to the "natural" diffraction pattern (without time delay changes). When we lower the angles to something like 18°, it becomes closer to the target, and the side lobes are lower as shown in Figure 6.

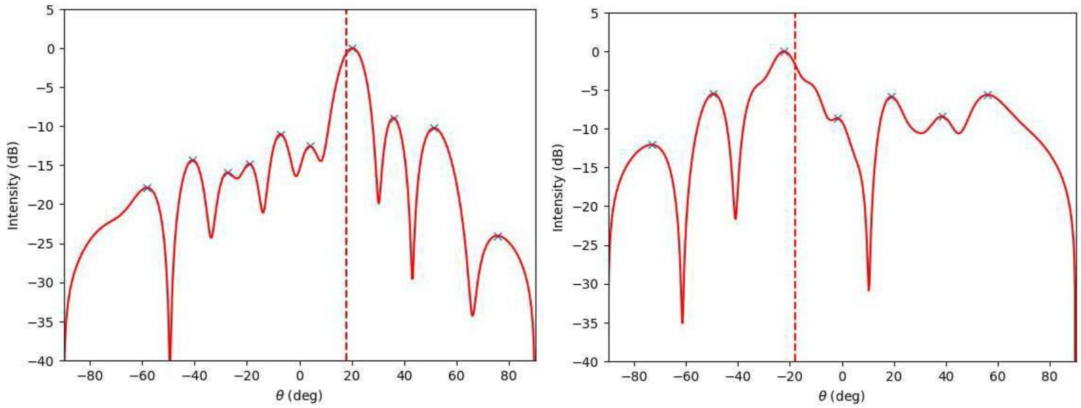

Figure 6: (a): Diffraction pattern of first harmonic (steered to +18°), (b): Third harmonic (steered to -18°), weight = 0.05

It is also possible to steer to different angles (not necessarily symmetric) as shown in Figure 7.

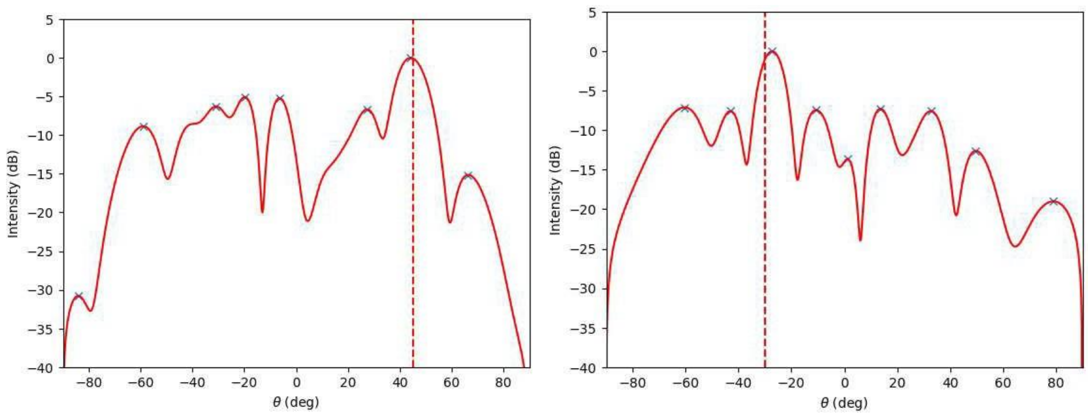

Figure 7: (a): Diffraction pattern of first harmonic (steered to +45°), (b): Third harmonic (steered to -30°), weight=0.05

With a low peak ratio weight, the side lobes are quite high. We can try to suppress them by increasing the weight, but this inevitably causes the steering to become poorer as evident from Figure 8.

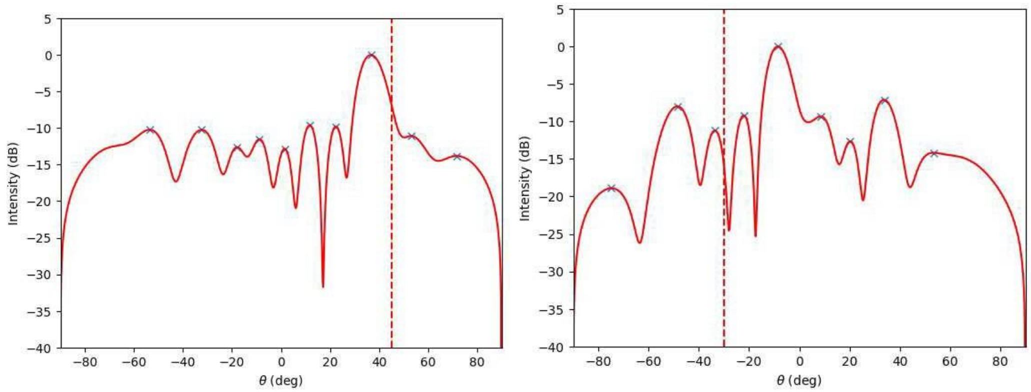

Figure 8: (a): Diffraction pattern of first harmonic (steered to +45°), (b): Third harmonic (steered to -30°), weight = 0.10

So far, we have only been changing the time delays, so that we can have independent control over the harmonic amplitudes. If we attempt to also vary the duty ratio, the effectiveness is limited, as there are too many quantities to vary. As seen in Figure 9, the steering is less optimal than the other solutions found by only varying the time delay (Figure 4(b)).

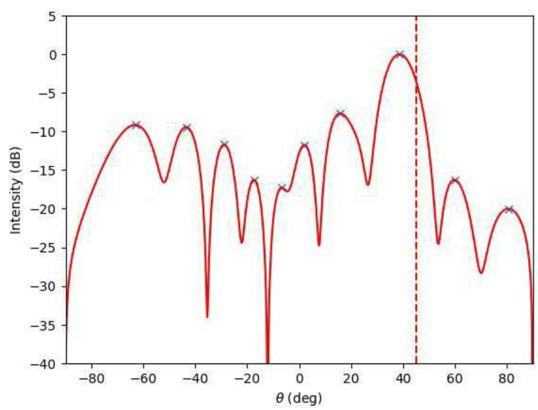

Figure 9: Diffraction pattern of first harmonic (steered to +45°) weight = 0.1

Discussion

We have concluded that the IPSO is best for optimising the metasurface, as it consistently performed the best in harmonic beam steering. Many applications of metasurfaces are similar to harmonic beam steering. Diffusion is essentially the opposite of beam steering as it incentivises the lack of a high primary maxima as opposed to a high primary maxima (as compared to the secondary maxima).

The unusual coding technique used in this paper holds much promise as the duty ratio can be changed to alter the ratio between the amplitudes of the various harmonics easily. As mentioned in the introduction, we could choose $MM = 0.5$ to suppress the fundamental harmonic and all even harmonics. Then if we change $MM = 2/3$, we suppress every third harmonic, while keeping the intensity distribution (across angles) within each harmonic the same.

The diffusion application can be rather useful as it masks the reflection of the metasurface by a considerable amount, reducing the maximum peak by 12dB, as compared to the maximum peak produced by an STM with equal duty ratio and no time delay. By changing the duty ratio, the fraction of energy within each harmonic can be adjusted.

Dual harmonic beam steering could also be useful as it allows information stored within the incident light (which could be, say, frequency modulated) to be transmitted to two sources at once. However, the ratio between the primary maxima and the next highest maxima is not very high (only -10dB in the first harmonic and -7dB in the third harmonic), and the steering precision is low, so its application may be limited.

Acknowledgements

I would like to thank Dr Chia for providing helpful guidance throughout this project.

References

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[2] L. Zhang, and T. J Cui, "Space-time-coding digital metasurfaces: principles and applications," AAAS Research, vol. 2021, 9802673.

Appendix 1: Derivation of Fourier series coefficients

The synthesis formula of the square wave is

$$\Gamma(tt) = \sum_{k=-\infty}^{\infty} aa_k ee^{ikw_0 t}$$

For $kk \neq 0$, we conduct Fourier series analysis:

$$aa_k = \frac{1}{TT_0}\int_0^{T_0} \Gamma(t)\, ee^{-ikw_0 t}\, pptt$$

$$= \frac{1}{TT_0}\left(\int_{-T_0/2}^{-/2} \Gamma_2\, ee^{-ikw_0 t}\, pptt + \int_{-/2}^{/2} \Gamma_1\, ee^{-ikw_0 t}\, pptt + \int_{/2}^{T_0/2} \Gamma_2\, ee^{-ikw_0 t}\, pptt\right)$$

$$= \frac{2}{TT_0}\left(\Gamma_1\left[-\frac{1}{\pi\pi kk\omega\omega_0} ee^{-ikw_0 t}\right]_{-/2}^{/2} + \Gamma_2\left[-\frac{1}{\pi\pi kk\omega\omega_0} ee^{-ikw_0 t}\right]_{/2}^{T_0/2} + \Gamma_2\left[-\frac{1}{\pi\pi kk\omega\omega_0} ee^{-ikw_0 t}\right]_{-T_0/2}^{-/2}\right)$$

$$= \frac{\pi(\Gamma_1 - \Gamma_2)}{2\pi\pi kkMM}\left(ee^{ikw_0 /2} - ee^{-ikw_0 /2}\right)$$

$$= rr_0 MM \frac{\sin \pi\pi kkMM}{\pi\pi kkMM}$$

$$= rr_0 MM \operatorname{sinc} \pi\pi kkMM$$

For $kk = 0$,

$$aa_k = \frac{1}{TT_0}\int_{-T_0/2}^{T_0/2} \Gamma(tt)\, pptt = MM\Gamma_1 + (1 - MM)\Gamma_2$$

Accounting for the time delay by replacing tt with $(tt - tt_0)$ in the synthesis formula and absorbing it into the Fourier series coefficient yields the expected coefficients.